Question 1. The regional manager of General Express is worried about the possibility of strikes by some of his employees. He has learned that the probability of strike by his pilots is 0.75 and the probability of strike by his drivers is 0.65. He also knows that if the drivers strike, then there is a 0.9 probability that the pilots will strike (in sympathy).

(a) What is the probability of both groups’ striking?

Solution: We are given $P(\text{Pilots}) = 0.75$, $P(\text{Drivers}) = 0.65$, $P(\text{Pilots} | \text{Drivers}) = 0.9$.

We have, $P(\text{Pilots} \cap \text{Drivers}) = P(\text{Pilots}) \times P(\text{Pilots} | \text{Drivers}) = 0.65 \times 0.9 = 0.585$.

(b) What is the probability that at least one group will strike?

Solution: $P(\text{Pilots} \cup \text{Drivers}) = P(\text{Pilots}) + P(\text{Drivers}) - P(\text{Pilots} \cap \text{Drivers}) = 0.75 + 0.65 - 0.585 = 0.815$.

Question 2. Police plan to enforce speed limits by using radar traps at 4 different locations within the city limits. The radar traps at each of the locations $L_1$, $L_2$, $L_3$ and $L_4$ are operated 30%, 20%, 10% and 40% of the time. A person who is speeding on his way to work has probabilities of 0.2, 0.3, 0.4 and 0.1 respectively, of passing through these locations. If a person received a speeding ticket, what is the probability that he/she was trapped on the second location $L_2$?

Solution: We are given the following probabilities: $P(L_1) = 0.2$, $P(L_2) = 0.3$, $P(L_3) = 0.4$, $P(L_4) = 0.1$ and $P(ST | L_1) = 0.3$, $P(ST | L_2) = 0.2$, $P(ST | L_3) = 0.1$, $P(ST | L_4) = 0.4$.

By Bayes’ theorem,

$$P(L_2 | ST) = \frac{P(ST | L_2)P(L_2)}{P(ST | L_1)P(L_1) + P(ST | L_2)P(L_2) + P(ST | L_3)P(L_3) + P(ST | L_4)P(L_4)}$$

$$= \frac{0.3 \times 0.2}{0.2 \times 0.3 + 0.3 \times 0.2 + 0.4 \times 0.1 + 0.1 \times 0.4} = \frac{6}{6 + 6 + 4 + 4} = \frac{6}{20} = 0.3$$

Question 3. A university examination consists of 100 questions. Each question has 5 possible answers, only one of them is correct. If a student who takes this exam answers questions by guessing what is the probability that he/she answers correctly at least 25 question?

Solution: Let $X$ count the number of correct answer among 100 answers. Then $X$ is a binomial random variable with parameters $n = 100$, $p = \frac{1}{5}$ and $q = \frac{4}{5}$. We have, $\mu = np = 20$ and $\sigma = \sqrt{npq} = 4$. Our aim is to evaluate $P(X \geq 25)$. Since $np = 20 > 5$, $nq = 80 > 5$ then

$$P(X \geq 25) \approx P(\text{normal} Y \geq 24.5)$$

$$= P\left(\frac{Y - \mu}{\sigma} \geq \frac{24.5 - 20}{4}\right) = P(Z \geq 1.13) = 0.5 - 0.3708 = 0.1292$$

Question 4. An aptitude test for pilots requires a series of operations to be performed quickly. Suppose that time needed to complete the test is normally distributed with mean 90 minutes and standard deviation 20 minutes.
(a) Find the probability that it takes a randomly selected pilot between 95 and 100 minutes to complete the test.

Solution:

\[ P(95 \leq X \leq 100) = P\left(\frac{95 - 90}{20} \leq Z \leq \frac{100 - 90}{20}\right) = P(0.25 \leq X \leq 0.5) = 0.1915 - 0.0987 = 0.0928 \]

(b) The top 5% of pilots get an honor certificate. How fast must a pilot complete the test to get the certificate?

Solution: We have to find the cut off-value \( z \) that separates 5% of the fastest pilots from the others. We have,

\[ 0.05 = P(X \leq z) = P\left(\frac{X - \mu}{\sigma} \leq \frac{z - 90}{20}\right). \]

Therefore,

\[ 0.45 = P\left(0 \leq Z \leq -\left(\frac{z - 90}{20}\right)\right). \]

that implies \( \frac{z - 90}{20} = -1.645. \) Hence, \( z = 90 - 20 \times 1.645 = 57.1. \)

It means that if a pilot completes his/her test in less than 57.1 minutes, he/she will obtain an honor certificate.

Question 5. Mark the following questions as "TRUE" or "FALSE":

(a) An event whose probability is greater than one is called a simple event. FALSE

(b) Events that cannot happen together are called independent. FALSE

(c) If a fair die is thrown twice, the probability that both times it will show 4 is \( \frac{1}{6} + \frac{1}{6} = \frac{1}{3}. \) FALSE

(d) The probability of success is always the same for each trial of a binomial experiment. TRUE